An Iterative Algorithm with Simultaneously Updating the Spectrum and the Image of Dual Energy Computed Tomography

Shaojie Chang, Xi Chen and Xuanqin Mou

***Abstract*—Knowing the X-ray spectrum is important to dual energy computed tomography (DECT) reconstruction. Whereas the spectrum is not always available in practice. In addition, the reconstructed image of DECT is extremely sensitive to noise which demands special noise suppression strategy in reconstruction algorithm design. Hence the iterative reconstruction methods by inducing regularization to elevate the image quality attract more attention. In this paper, we develop an iterative algorithm with simultaneously updating the spectrum and the image of DECT, which did not need the knowledge of the spectrum in advance. Namely, spectrum estimation and iterative reconstruction are incorporated in an iterative framework. The estimated spectrum and the reconstructed images are obtained simultaneously with the proposed algorithm. Two sets of simulation experiment with different phantoms were adopted for evaluation. In the experiment, the spectrum estimated by the image based reconstruction results are utilized as the initially estimated spectrum. In comparison with the initially estimated spectrum, experimental results validated that the proposed method can increase the accuracy of the estimated spectrum. Besides, the quality of the reconstructed density images has been improved by this work at the same time.**

***Index Terms*—DECT, spectrum estimation, iterative reconstruction.**

I. Introduction

DUAL energy computed tomography (DECT) plays an important role in material characterization, bone mineral density inspection, nondestructive evaluation and so on[1–3]. For DECT, two sets of projection under high and low energy are acquired. By taking advantages of the projections, energy-selective images, images of the electron density and the effective atomic number, or material-selective images can be reconstructed. For decades, existed DECT reconstruction methods are able to be classified i n to t h ree c a tegories: projection based methods[4], image based methods[5, 6] and iterative methods[7–9]. With regard to projection based methods, projection measurements are split into two sets of projections of basis materials by using a nonlinear model[4]. Afterwards, two basis material images are reconstructed from the decomposition projections by conventional reconstruction algorithms. For image based methods, the basis material density images are obtained by means of a linear combination of the reconstructed images at two selected energy levels.[5]. In consideration of the statistical properties of noise contained in the raw data, several iterative reconstruction algorithms have been proposed.

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The special noise suppression strategies in iterative algorithm were designed to reduce the noise, such as total variation (TV) regularization[10], dictionary learning regularization[11] and so on. J.A.Fessler *et al.* investigated a maximum-likelihood method for dual energy tomography reconstruction[8]. Afterwards, Xu *et al.* presented a penalized-likelihood iterative reconstruction algorithm for polychromatic dual-energy CT[9]. Moreover, in DECT iterative reconstruction, the knowledge of X-ray spectrum are needed. As a result, how to acquire the x-ray spectrum information of DECT is also exceedingly imperative.

In practice, it is sometimes difficult to acquire the spectrum directly. As a consequence, a lot of indirect spectrum estimation algorithms have been suggested. In these algorithms, the spectrum is mostly reconstructed from attenuation data, which is produced by a phantom with known dimensions and compositions. For example, E.Y.Sidky *et al.*[12] applied the expectation-maximization (EM) method to estimate the spectrum from transmission data. However, the EM method could reconstruct the accurate spectrum only if the initial guess value is close to the true value in the iteration schedule. And it could not recover the details of the characteristic spectrum of x-ray. Another spectrum estimation method is based on the x-ray physical model. Yang *et al.*[13] proposed a seven parameters model based spectrum reconstruction algorithm. And there consist three parts of the proposed model: bremsstrahlung spectrum, photoemission attenuation by x-ray tube inherent filter and characteristic radiations. The proposed method delivered a perfect reconstruction of the spectrum by solving the parameters. To avoid calculating each energy bin of the spectrum, Zhao *et al.* [14] stated that an estimated spectrum can be expressed as several known spectrum models with different weights, which exploited the difference between the raw projection data of a phantom with known density and the projection with estimated spectrum to calculate the weights corresponding to the spectrum models. But the proposed method needed segmentation to generate the estimated projection. Afterwards, based on the previous work, Zhao *et al.* [15] developed a segmentation-free indirect transmission measurement-type energy spectrum estimation method by using dual-energy material decomposition, which needed the noise reduced significantly material-specific images. From the above, the spectrum could be reconstructed by several methods, but the influence of the estimated spectrum utilized in the CT image reconstruction problem has not been further investigated and the spectrum estimation algorithms are independent of DECT image reconstruction.

Motivated by this, an iterative algorithm with simultaneous-

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ly updating the spectrum and the image of dual energy computed tomography is demonstrated. In this work, the high and low energy spectrum of DECT are not obliged to give in advance. Built on the model spectra, we develop a polychromatic projection model to acquire the estimated projection. By utilizing the difference between the raw projection and the estimated projection, the weights of the corresponding model spectra will be computed to generate the estimated spectrum. Furthermore, we applied them into the DECT iterative reconstruction. The estimated spectrum and reconstructed density images will be simultaneously updated in each iteration. The contribution of this work is twofold. First, for DECT, the knowledge of the spectrum is no longer required to acquire beforehand. Second, based on the proposed method, the accurate estimated spectra and noise reduced reconstructed images can be simultaneously obtained.

The rest of the paper is organized as follows: Section II presents the key idea in details. In Section III, experimental results are illustrated to validate the proposed method. Section IV expressed the discussion of this study. In section V, we conclude with a summary.

II. Methods and Materials

A. DECT Spectrum Estimation Based on Model Spectra

Based on Zhao's work[14, 15], instead of estimating each energy bin content for a spectrum $\mathrm{S(E)}$, a weighted summation of a set of model spectra $\mathrm{S_m(E)}$ is utilized to describe $\mathrm{S(E)}$,

$$\mathrm{S(E)} = \sum_{\mathrm{m=1}}^{\mathrm{M}} \mathrm{c_m S_m(E)} \tag{1}$$

where M is the number of model spectra, $\mathrm{c_m}$ represents the weight on the corresponding spectrum. $\mathrm{S_m(E)}$ describes the model spectrum with the different lengths of the tube filter. Based on the model spectra, omitting scatter photons, the discrete form of DECT polychromatic projections is modeled as follows[9][16]:

$$\mathrm{p_{k,j}} = \sum_{\mathrm{i=1}}^{\mathrm{N}} \mathrm{S(E_i)} \exp -(\mathbf{R_j} \mathrm{m_w(E_i)} \boldsymbol{\rho}_{\mathrm{w}} + \mathbf{R_j} \mathrm{m_b(E_i)} \boldsymbol{\rho}_{\mathrm{b}}) \tag{2}$$

where k denotes high and low X-ray tube energy in DECT, j represents the $\mathrm{j}th$ X-ray path. N stands for the number of sampling intervals of X-ray energy spectrum and i is corresponding to the $\mathrm{i}th$ sampling interval. $\mathrm{S(E_i)}$ represents $\mathrm{i}th$ normalized effective spectrum, which could be substituted by (1). And let $\mathbf{R_j}$ be forward projection vector of $\mathrm{j}th$ ray. $\mathrm{m_w(E_i)}$ and $\mathrm{m_b(E_i)}$ represent the mass attenuation coefficients of water and bone at the photon energy $\mathrm{E_i}$, respectively. Denoting $\boldsymbol{\rho}_{\mathrm{w}}$ and $\boldsymbol{\rho}_{\mathrm{b}}$ as the density image of water and bone.

As we all know, two sets of projections under high and low energy are provided by DECT. By utilizing image based reconstruction algorithm of DECT, two known material density images $\boldsymbol{\rho}_{\mathrm{w}}^0$ and $\boldsymbol{\rho}_{\mathrm{b}}^0$ are obtained as the initial density images. And then the estimated projection $\widetilde{\mathbf{p}}$ can be formed with the model spectra and the known density images by (3).

$$\widetilde{\mathbf{p}}_{\mathrm{k,j}} = \sum_{\mathrm{i=1}}^{\mathrm{N}} \sum_{\mathrm{m=1}}^{\mathrm{M}} \mathrm{c_m S_m(E_i)} \exp -(\mathbf{R_j} \mathrm{m_w(E_i)} \boldsymbol{\rho}_{\mathrm{w}} + \mathbf{R_j} \mathrm{m_b(E_i)} \boldsymbol{\rho}_{\mathrm{b}}) \tag{3}$$

Note that the difference between estimated projections $\widetilde{\mathbf{p}}$ and projection measurements $\mathbf{p}$ should be minimal if the estimated spectrum matches the ideal spectrum. Afterwards, the problem can be expressed as follows,

$$\mathbf{c} = \underset{\mathbf{c}}{\mathrm{argmin}} \, \|\widetilde{\mathbf{p}} - \mathbf{p}\|_2^2, \mathrm{s.t} \sum_{\mathrm{m=1}}^{\mathrm{M}} \mathrm{c_m} = 1, \mathrm{c_m} > 0. \tag{4}$$

For seeking an appropriate parameter vector $\mathbf{c}$, we apply a BFGS method[17] by optimizing the objective function (4). And the initial values of $\mathbf{c}$ are set to equal. In the function, the normalized constraint $\sum_{\mathrm{m=1}}^{\mathrm{M}} \mathrm{c_m} = 1$ and the non-negative constraint are introduced. Iteratively loop will not be terminated until the difference between $\widetilde{\mathbf{p}}$ and $\mathbf{p}$ is smaller than ε, where ε is set to 10^{-6}. Finally, the high and low energy spectrum are estimated by solving the above problem.

B. Statistic Iterative Reconstruction Algorithm of DECT

With the estimated spectrum, a statistic iterative reconstruction method is utilized to suppress image noise by introducing ℓ_1-norm based regularization for polychromatic DECT. Firstly, in this work, we assumed that the projection measurements $\mathbf{p}$ follow Poisson statistical model. To reduce the noise in the reconstructed density images, a ℓ_1-norm based regularization combined with the log-likelihood is employed in iterative reconstruction. Here, the Total Variation (TV)[10] regularization is chosen. Consequently, the objective function is expressed as:

$$\Phi(\rho) = \mathrm{L}(\rho) - \lambda_{\mathrm{TV}} \mathrm{TV}\,(\rho) \tag{5}$$

where $\mathrm{L}(\rho)$ represents the Poisson log-likelihood for polychromatic DECT. ρ is a pair of two basis material density images. $\mathrm{TV}\,(\rho)$ is a regularization term removing the noise of the material density images ρ whose strength is controlled by λ_{TV}. The reconstruction images are obtained by maximizing the objective function (5). The well-known convex technique, gradient ascent algorithm and ordered subsets acceleration are employed to optimize the above problem. Owing to the limited space, details are not presented here.

C. An Iterative Algorithm with Simultaneously Updating The Spectrum and The Image

In Zhao's work[15], it is suggested that the spectrum estimation algorithm based on model spectra is strongly relied on the original basis material density images. That is to say, the original images contained noise probably affect the accuracy of the estimated spectrum. In addition, the reconstruction results will also be influenced when the estimated spectrum is induced into the iterative reconstruction. However, it is worth emphasizing that on the basis of our previous study[18], the proposed statistic iterative reconstruction can reduce the noise significantly in reconstructed images with the estimated spectrum. A natural idea is that we can incorporate the iterative reconstructed images and the spectrum estimation algorithm in an iterative scheme to increase the accuracy of the estimated spectrum and the quality of the reconstructed images. Motivated by this, an iterative algorithm with simultaneously updating the spectrum and the image of DECT is proposed. The flowchart is shown in Fig.1.

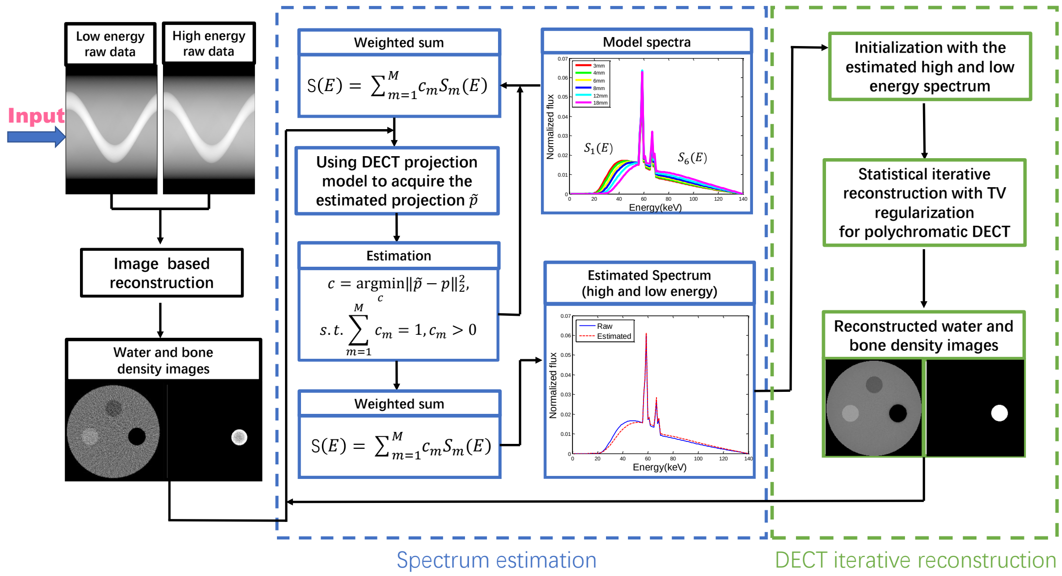

Fig. 1. The flowchart of the proposed algorithm with simultaneously updating the spectrum and the image for DECT.

The proposed algorithm starts from the given high and low energy projection data scanned by DECT. Two original material density images can be reconstructed by an image based reconstruction algorithm. Next, known density images and the model spectra are employed to generate the estimated projections by the DECT projection model. To acquire the corresponding weights of the model spectrum, we should optimize the function of the difference between the estimated projection and the raw data. In this work, the spectrum estimated by the original density images are defined as the initially estimated spectrum. Afterwards, the estimated spectrum are constituted into the statistical iterative reconstruction for polychromatic DECT. In order to achieve a more accurate estimated spectrum, we introduce the reconstructed results to the spectrum estimation algorithm. In each iteration, the estimated spectrum and reconstructed images are simultaneously updated. The iteration will not be terminated until the RMSE of the reconstructed results remains stable. Finally, the estimated spectrum of DECT and the two basis material density images are obtained at the same time.

D. Experiments

In this paper, simulation experiments are adopted to verify the proposed method. We employ two phantoms to reveal the effectiveness of the proposed method. One is a physical phantom, in which there exist three target disks (density: $0.95\mathrm{g/cm^3}$, $1.05\mathrm{g/cm^3}$ and $1.8\mathrm{g/cm^3}$, respectively) placed in water disk (density: $1.0\mathrm{g/cm^3}$). And another is a 2-D slice of abdomen density phantom designed by Jeffery A.Fessler[19]. The phantom size is 256×256.

For simplicity, the numerical experiments were restricted to equi-spatial fan beam CT. The scanning configuration of a dual source CT scanner was set as follows: the distance from the x-ray source to the system origin was $1000\mathrm{mm}$, and the distance from the source to detector was $1200\mathrm{mm}$. Totally, 720 projections are collected over a full scan range. The detector is composed of 1024 detector elements of $0.3\mathrm{mm}$ per element.

In our work, $140\mathrm{kVp}$ and $80\mathrm{kVp}$ are chosen as the high and low tube energy of the dual energy CT. The generator of the model spectra is the SpectrumGUI software [see http://spectrumgui.sourceforge.net/]. The model spectra could also be predetermined by other spectrum generators, such as SpekCalc and Spektr. In addition, the number of model spectra is 6, which has been approved in Zhao's work[14]. We choose Al as the material of the tube filter. And the lengths of Al in model spectra are 3mm, 4mm, 6mm, 8mm 12mm and 18mm respectively.

In this study, 1×10^6 incident photons are superimposed into noise-free sinograms to synthesize projection measurements. Subsequently, the estimated spectrum and the reconstructed images can be acquired. We denote the spectrum estimated with the image based reconstruction results as the initially estimated spectrum. And it is utilized to compare with the spectrum estimated by the proposed method. To quantify the accuracy of the estimated spectrum, the normalized root mean square error (NRMSE) is calculated between the estimated spectrum and the original spectrum as follows,

$$\mathrm{NRMSE} = \sqrt{\frac{\sum_{i=1}^{M} (\widetilde{\mathrm{S}}(\mathrm{E}) - \mathrm{S}(\mathrm{E}))^2}{\sum_{i=1}^{M} (\mathrm{S}(\mathrm{E}))^2}} \tag{6}$$

The reconstructed results of the two phantoms are displayed. For the further comparison, we calculate the root mean square error (RMSE) of reconstruction images in contrast to the ground truth to demonstrate the denoising performance of the proposed method. Moreover, to evaluate the influence of the accuracy of the estimated spectrum, the images quality reconstructed with the initially estimated spectrum and the proposed method will be compared. To depict the efficiency of the proposed method clearly, we plot a profile of the images reconstructed with each estimated spectrum.

III. Results

In Fig. 2, the spectrum estimated by the proposed method with different phantoms are illustrated. The estimated spectrum are compared with the original spectrum and the initially

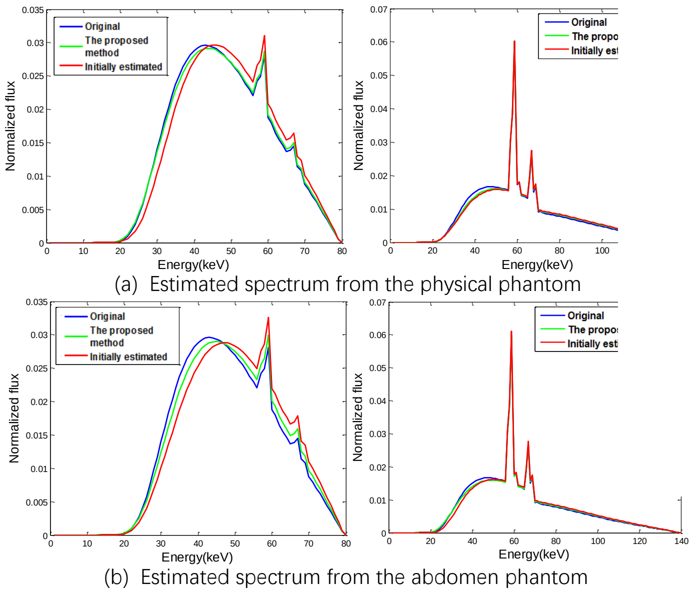

Fig. 2. The high and low tube energy estimated spectrum of the proposed method by different phantoms. (a) and (b) show the estimated spectrum by the physical phantom and the abdomen phantom. The blue line represents the original spectrum and the red line describes the initially estimated spectrum, respectively. And the green line shows the estimated spectrum by proposed method.

TABLE I
THE NRMSE BETWEEN ORIGINAL SPECTRUM AND ESTIMATED SPECTRUM

Tube Energy	The Proposed Method	Initially Estimated Spectrum
Low tube energy	6.01%	14.3%
High tube energy	3.51%	6.41%

estimated spectrum. Without regard to the scanned object, it is apparent that the proposed method delivers an estimated spectrum close to the original spectrum at high and low tube energy. To evaluate the performance of the proposed method in abdomen phantom, the NRMSE of each estimated spectrum is calculated. The results shown in Table I demonstrate that the proposed method increases the accuracy of the estimated spectrum. The NRMSE for the low tube energy spectrum of the proposed method is 6.01%, while the value of the initially estimated spectrum is 14.3%.

Figure. 3 illustrates the reconstructed results o toms with 10^6 photons. It is suggested that th method offers a superior denoising performance f contrast with the image based reconstruction result verify the denoising performance, the RMSE of compared with the ground truth are manifested The RMSE indicates that the noise of the propo has been intensely eliminated.

In Fig. 4, the image quality of the proposed better than the image reconstructed with the initial spectrum visually. Furthermore, the profile result our work can deliver an accurate reconstructic contrast with the results reconstructed with the i mated spectrum. In other words, a more accurate re image is obtained with significant estimated spectr improvement.

IV. Discussion

The x-ray spectrum plays a crucial role in DECT iterative reconstruction, which has not always been available in practice.

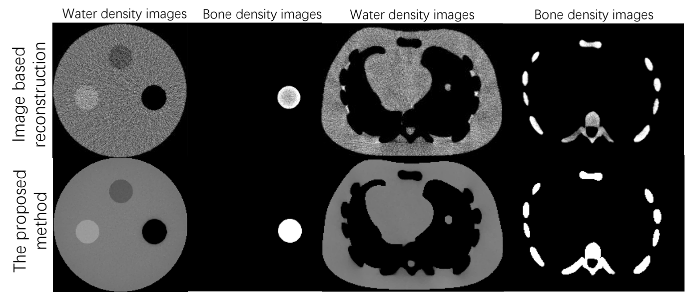

Fig. 3. The reconstructed density images of the image based reconstruction and the proposed method with two phantoms. The top row shows the results reconstructed by the image based reconstruction method and the bottom presents the results of the proposed method, respectively. Water density images are shown in gray scale window [0.8,1.2] and bone density images are displayed in gray scale window [1.4,1.8].

TABLE II
THE RMSE OF RECONSTRUCTED RESULTS IN CONTRAST WITH THE GROUND TRUTH

Phantom	Image Based Reconstruction	The Proposed Method
Physical Phantom	2.08×10^{-3}	3.19×10^{-4}
Abdomen Phantom	2.9×10^{-3}	2.12×10^{-3}

The proposed algorithm can obtain the estimated spectrum and reconstructed images at the same time. The accuracy of the estimated spectrum could affect the image quality of the reconstructed images. However, there exists no quantitative analysis on the problem presented so far. As a result, it is essential to explore a particular study on the influence of the estimated spectrum accuracy for DECT reconstructed images in our future work.

As stated before, the efficacy of the proposed algorithm has been proved in this work, while the scatter effect is not taken into account. The scatter effect in spectrum estimation and image reconstruction in DECT will be also an attractive problem in the future. Therefore, our goal is to develop a unified DECT statistical iterative reconstruction framework

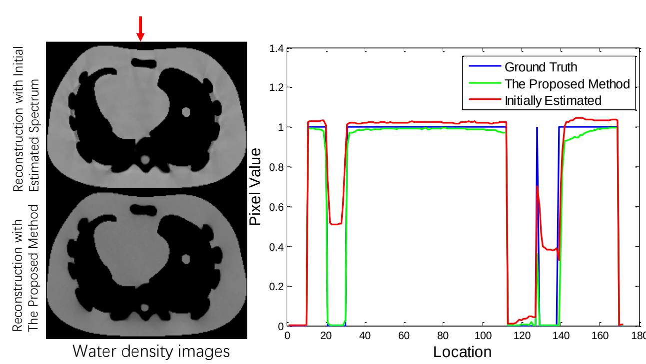

Fig. 4. The water density image quality reconstructed with the initially estimated spectrum and the proposed method. The left column showed the results reconstructed by the initially estimated spectrum and the proposed method, respectively. The right part displayed the profile generated along the red arrow, which labeled in the water density images. The blue line represented the ground truth. The green line and the red line denoted the results with the initially estimated spectrum and the proposed method, respectively. Water density images are shown in gray scale window [0.8,1.2].

spectrum estimation, denoising, reconstruction and scatter radiation calculation together in iterative reconstruction.

Besides, the proposed method is only confirmed on the simulation experiments. For a good prospect of application, a multiple of real data under practical DECT scanning will be adopted to validate the effectiveness and stableness of the proposed method.

V. Conclusion

In this paper, we develop an iterative algorithm with simultaneously updating the spectrum and the image of dual energy computed tomography. The key idea is that spectrum estimation and image reconstruction are incorporated in one iteration to increase the accuracy of the results. The proposed method only needs the given projection measurements of DECT. And then the estimated spectrum and reconstructed images can be acquired at the same time. The efficiency and robustness of the proposed method have been confirmed by experimental results. To sum up, the proposed iterative algorithm can increase the accuracy of the estimated spectrum and significantly improve the quality of the reconstructed images.

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